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SEMI PRIME DERIVATION ALTERNATOR RINGS

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ABSTRACT: In [4] Rich showed that a prime ring with idempotent $e \neq 1$ and if every idempotent lies in their nuclei, then R is alternative. In this section we prove that a semiprime derivation alternator ring with idempotent $e\neq1$ and characteristic $\neq 2$, then idempotent is in flexible nucleus. At the end of this section we give some examples.

Index Terms: Derivation alternator ring, Nucleus, Prime ring, Semi prime ring, Flexible nucleus.

1. INTRODUCTION:

In [1] and [2] E.Kleinfeld defined two different generalizations of alternative rings, and for each generalizations he showed that the simple rings with idempotent $e\neq1$ to be alternative. Both of these generalizations defined by kleinfeld are contained in the derivation alternator rings, and this we extend his results to simple derivation alternator rings with idempotent $e\neq1$ and characteristic $\neq2$. Hentzel and Smith [3] investigated the structure of non-associative, flexible derivation alternator rings.

A non-associative ring with characteristic$\neq2$ is called a derivation alternator ring if it satisfies the identities:

$,x,x)=0$ 1

$(yz,x,x)=y(z,x,x)+(y,x,x)z$ 2

$(x,x,yz)=y(x,x,z)+(x,x,y)z$ 3

Linearizing (1) leads to the identity

$(x, x, y) + (x, y, x) + (y, x, x) = 0$ 4

Now linearize equation (2) gives

$(yz,x,w)=y(z,x,w)+(y,x,w)z$ 5

replace y=z=x,x=y and w=z we have

$(x^2,y,z)=x(x,y,z)+(x,y,z)x$

$(x^2,y,z)=xo(x,y,z)$ 6

where $xoy=xy+yx$.

Linearize equation (3) we have

$(w,x,yz)=y(w,x,z)+(w,x,y)z$ 7

Replace y=z=x,x=y,w=z we have

$(z,y,x^2)=x(z,y,x)+(z,y,x)x$

$(z,y,x^2)=xo(z,y,x)$ 8

Replacing x by x + w in (6) and (8) yields

$(x \text{ o } w, y,z) = x \text{ o } (w, y, z) + w \text{ o } (x, y, z)$ 9

and

$(z, y, x \text{ o } w) = x \text{ o } (z, y, w) + w \text{ o } (z, y, x)$. 10

The Teichmuller identity holds for all rings

$(wx,y,z)-(w,xy,z)+(w,x,yz)=w(x,y,z)+(w,x,y)z$ 11

Also the following identity holds in any ring

$[xoy,z]+[yoz,x]+[zox,y]=0$ 12

In this paper, let A denote derivation alternator ring satisfies (1) to (3).

We define the flexible nucleus of a non-associative ring A to be

$N_F(A)=\{r\in A/ 0=(x,r,x)=(r,x,r)=(r,x,y)+(y,x,r)$ for all $x,y\in A\}$.

2.Main Results:

LEMMA1: Let $H =\{ h\in A_{1/2} / [e, h]=0 \}$.Then $HA_i\subseteq H$ and $A_iH\subseteq H$ fori=0,1.

PROOF: Let $h\in H$ and $x_i\in A_i$ for i=0,1.

From (2) and the definition of H we have

$0= ([e,h],w,w)=[(e,w,w),h]+[e,(h,w,w)]$ for all $w \in A$.

In particular,

$0=[(e,e,x_i)+(e,x_i,e), h]+[e, (h,e,x_i)+(h,x_i,e)]= [e, (h,e,x_i)+(h,x_i,e)]$

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since $(h,e,x_i)+(h,x_i,e)\in A_{1/2}$, it follows
$(h,e,x_i)+(h,x_i,e)\in H$.
Now h=2he, so
$2[(h,e,x_i)+(h,x_i,e)] = (1-4i)hx_i +2(hx_i)e \in H$.
Hence we obtain

$$(4i-1)hx_i \equiv 2(hx_i)e \mod H. \qquad 13$$

Then, since $0=[e,h]=2[e,he]$ implies $He \subseteq H$, multiplying (13) through on the right by 2e gives
$(8i-2)(hx_i)e \equiv 4[(hxi)e]e \mod H$.
But using (2) and $4(h,e,e)=2he-4he=-h$, we have

$$4[(hx_i)e]e = 4(hx_i,e,e)+4(hx_i)e$$
$$=4[h(x_i,e,e)+ (h,e,e)\, x_i]+ 4(hx_i)e$$
$$=-hx_i+4(hx_i)e.$$

Thus we arrive at
$(8i-2)(hx_i)e \equiv -hx_i+4(hx_i)e \mod H$, or

$$hx_i \equiv (3-4i)2(hx_i)e \mod H. \qquad 14$$

Finally combining (13) and (14) leads to mod H to

$$hx_i \equiv (3-4i)2(hx_i)e$$
$$\equiv (3-4i)(4i-1)hx_i =-3hx_i \text{ for } i= 0 \text{ or } 1.$$

But then $4hxi \equiv 0 \mod H$, so that $HA_i \subseteq H$.
Similarly we can obtain $A_iH \subseteq H$.
Hence lemma is proved.♦

LEMMA 2: $(e,x,y_{1/2})+(y_{1/2},x,e)\in Z$ for $y_{1/2} \in A_{1/2}$ and $x\in A$.
PROOF: From equation (6) and (8)we have

$$(e^2, x,y_{1/2})=eo(e,x,y_{1/2}) \qquad 15$$
$$(y_{1/2},x,e^2) =eo(y_{1/2},x,e) \qquad 16$$

By adding (15) and (16) we have
$(e, x,y_{1/2})+ (y_{1/2},x,e) =eo[(e,x,y_{1/2})+ (y_{1/2},x,e)]$
Also subtracted one equation from another, we obtain
$[e, (e,x,y_{1/2})+ (y_{1/2},x,e)]=0$.
In particular this shows that

$$(e,x,y_{1/2})+ (y_{1/2},x,e) \in H. \qquad 17$$

Next let $x_{1/2}\in A_{1/2}$. Then the fact $x_{1/2}oy_{1/2}\in A_1+A_0$ in the Albert decomposition [5], we have
$(e,x_{1/2},y_{1/2})+(y_{1/2},x_{1/2},e) = (ex_{1/2})y_{1/2}- e(x_{1/2}y_{1/2})+(y_{1/2}x_{1/2})e-y_{1/2}(x_{1/2}e) = (ex_{1/2})oy_{1/2}- e(x_{1/2}oy_{1/2})+(y_{1/2}x_{1/2})oe-y_{1/2}x_{1/2} \in A_1+A_0$
But this together with (17) implies

$$(e, x_{1/2},y_{1/2})+ (y_{1/2}, x_{1/2}, e)\in (A_1+A_0) \cap A_{1/2} =0. \qquad 18$$

Let $x_i\in A_i$ for i=0, 1, from linearized (7), theorem (2) and (18) we have
$[(e,x_i,y_{1/2})+(y_{1/2},x_i,e)]x_{1/2} = [(e,x_ix_{1/2},y_{1/2})+(y_{1/2},x_ix_{1/2},e)]- x_i[(e,x_{1/2},y_{1/2})+(y_{1/2},x_{1/2},e)] = 0.$
Thus in conjunction with (18) we have
$[(e,x,y_{1/2})+(y_{1/2},x,e)]=0$.
Similarly we go for opposite way we get

$$A_{1/2}[(e,x,y_{1/2})+(y_{1/2},x,e)]= 0 = [(e,x,y_{1/2})+(y_{1/2},x,e)]\, A_{1/2} \qquad 19$$

Lastly, let $h = [(e,x,y_{1/2})+(y_{1/2},x,e)]$ and again $x_i\in A_i$ for i=0,1.then by (12) we have
$0=[eox_i, h]+[x_ioh, e]+[hoe, x_i] = (2i-1)[x_i,h]+[x_ioh,e]$.
But by (17) $[x_ioh,e]=0$ and lemma (13),we have
$(2i-1)[x_i, h]= 0$.
Hence we obtain

$$[A_i, (e,x,y_{1/2})+(y_{1/2},x,e)] =0 \text{ for } i=0,1. \qquad 20$$

Hence lemma proved.♦

THEOREM 1: If A is a semiprime derivation alternator ring with idempotent $e\neq 1$ and characteristic $\neq 2$, then idempotent e is in flexible nucleus.
PROOF: It will suffice to show $(e,x,y)+(y,x,e) =0$ for all $x,y\in A$, since then also $(x,e,x)=-\{(e,x,x)+(x,x,e)\}=0$ by linearized (1).
Since R is semiprime, from (14) we have

$$(e,x,y_{1/2}) + (y_{1/2}, x, e) =0 \text{ for } x \in A \text{ and } y_{1/2}\in A_{1/2}. \qquad 21$$

Also from theorem (2) we have

$$(e, x_j,y_i) = 0 = (y_i, x_j, e) \text{ for } x_j\in A_j \text{ and } y_i\in A_j \text{ where } i,j=0,1. \qquad 22$$

We next consider $x\in A_{1/2}$ and $y_i\in A_i$ for i=0,1.
From (6) and (8) put z=x and by linearizing, we have
$(e^2,x,y_i) = eo[(e,x,y_i)+(y_i,x,e)]$
And $(y_i,x,e^2) = eo[(e,x,y_i)+(y_i, x,e)]$
Since $w = (e,x,y_i)+(y_i,x,e)\in A_{1/2}$ from theorem(2), adding last two equations gives
W=2eow=2w. Hence w=0, so that

$$(e,x,y_i)+(y_i,x,e)=0 \text{ for } x\in A_{1/2} \text{ and } y_i\in A_i \text{ where } i=0,1. \qquad 23$$

From equations (21),(22) and (23) that $(e,x,y)+(y,x,e) =0$ for all $x,y\in A$ this completes the theorem.♦

Example 1: Let A be any Lie ring. Then, since $x^2=0$ and $xoy=0$ for all x, y□A, the identities (1), (6), and (8) must hold in A. Hence, there are simple finite-dimensional nil algebras satisfying (1), (6), and (8) (the sample Lie algebras), so that postulating the existence of an idempotent severely limits the possibilities for A when A is simple.

Example 2: A direct calculation shows that the algebra A of class which is given by the basis {1, x, y} where $x^2 = y^2 = 0$, xy=-yx=1 satisfies (1), (6) and (8). Thus A is a derivation alternator ring. In [6] kleinfeldet. defined a construction which gave rise to a class to simple finite-dimension algebras satisfying the identify (x, y, z) = (z, y, x), in which the flexible identify (x, y, x) =0 fails. Hence, these algebras (which possess unity elements) cannot be alternative.

Example 3: Let A be an algebra over the field F with a basis {e, x, y} where e^2 =e, ex=x + y, xe = -y, ey=y, ye=$x^2 = y^2$ = xy = yx = 0. We see that A_1 = Fe, $A_{1/2}$ = Fx + Fy, A_0 = 0. If z=□e + □x+□y, z= □,□,□□F then z^2 =□z so that A is power-associative and satisfies (1). Any easy calculation reveals that (w, u, v)□$A_{1/2}$ for all w, u,□v A. But then (z^2, u, v)=(□z,u,v) = □(z,u,v) while (z, u, v) = zo (z, u, v) and (3.1.6) holds. In a similar fashion (3.1.8) must be valid in A. We see that e(xe) = (ex)e =-y □0 so that $A_{1/2}$ does not decompose into $A_{10} + A_{01}$.

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